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Edge Product Cordial Labeling of Some Special Graphs

[1]V. Maheswari, [2]B.Vijaya Praba, [3]K.Mahalakshmi
[1] Assistant Professor, [2,3] PG Mathematics
PG & Research Department of Mathematics
A.P.C. Mahalaxmi College for Women, Thoothukudi

Abstract

The concept of edge product cordial labeling was introduced by S.K.Vaidya and C.M.Barasara. For a graph G, the edge labeling function is defined as $f: E(G) \to \{0,1\}$ and induced vertex labeling function $f^*: V(G) \to \{0,1\}$ is given as if $e_1, e_2, \dots, e_n$ are the edges incident to the vertex v, then $f^*(v) = f(e_1)f(e_2) \dots f(e_n)$. Let us denote $v_f(i)$ is the number of vertices of G having label i under f^* and $e_f(i)$ is the number of edges of G having label i under f for $i = 0, 1$. f is called edge product cordial labeling of G if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is called edge product cordial if it admits edge product cordial labeling. In this paper, we investigate the edge product cordial labeling of some special graphs.

Keywords: Edge product cordial labeling, Edge product cordial graphs.

2000 AMS Subject Classification: 05C78

1. Introduction

Graph labeling was introduced by Alexander Rosa in the year 1967. Rosa identified three types of labeling which was later renamed by Solomon Golomb. If the vertices are assigned values subject to certain condition(s) then it is known as *graph labeling*. A mapping $f: V(G) \to \{0,1\}$ is called *binary vertex labeling* of G and $f(v)$ is called *label* of vertex v of G under f.The concept of cordial labeling was introduced by Cahit [1] in 1987 and in the same paper he investigated several results on this newly introduced concept. A latest survey on various graph labeling problems can be found in Gallian [2]. Motivated through the concept cordial labeling, M.Sundaram, R.Ponraj and S.Somasundaram [6] have introduced a labeling which has the flavour of cordial labeling but absolute difference of vertex labels is replaced by product of vertex labels. The concept of edge product cordial labeling was introduced by S.K.Vaidya and Barasara [7].

Definition: 1.1

A *graph labeling* is an assignment of integers to the vertices or edges or both subject to certain condition(s). If the domain of the mapping is the set of vertices (or edges) then the labeling is called *a vertex labeling (or an edge labeling).*

Definition: 1.2

A binary vertex labeling of graph G is called a *cordial labeling* if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is called *cordial* if it admits a cordial labeling.

Definition: 1.3

A binary vertex labeling of graph G with induced edge labeling $f^*: E(G) \to \{0,1\}$ defined by $f^*(e = uv) = f(u)f(v)$ is called a *product cordial labeling* if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is *product cordial* if it is admits a product cordial labeling.

Definition: 1.4

For graph G, the edge labeling function is defined as $f: E(G) \to \{0,1\}$ and induced vertex labeling function $f^*: V(G) \to \{0,1\}$ is given as if $e_1, e_2, \dots, e_n$ are the edges incident to the vertex v then $f^*(v) = f(e_1)f(e_2) \dots f(e_n)$.

Let us denote $v_f(i)$ is the number of vertices of G if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is called *edge product cordial* if it admits edge product cordial labeling.

Definition: 1.5

A *fan* is a graph obtained from a path $P_n$by joining each vertices of P_n to a pendent vertex, it is denoted by $F_n = P_n + K_1$

Definition: 1.6

The *friendship graph* fn is a collection of$n$triangles with a common vertex. It may be also pictured as a wheel with every alternate rim edge removed.

The Generalised friendship graph $f_{q,p}$is a collection of p cycles (all of order q), meeting at a common vertex.

Definition: 1.7

The *sunlet graph* S_n is the graph obtained from a cycle C_n attaching a pendant edge to each vertex of the $n -$ cycle.

Definition: 1.8

The graph obtained from a path attaching exactly two pendant edges to each internal vertex of the path is called *Twig* and it is denoted by $T(n)$.

Definition: 1.9

A graph obtained from a path P_n by attaching a pendant edge to every internal vertices of the path is called *Hurdle graph* with n-2 hurdles and is denoted by Hd_n.

2.Main Results

Theorem 2.1 The graph F_n is edge product cordial graph for even n and not an edge product cordial graph for odd n.

Proof: Let F_n be a fan graph $F_n = P_n + K_1$where $P_n$is the path for even n and K_1 is the complete graph with one vertex.

Let $e_1, e_2, \dots, e_{2n-1}$ be the successive edges of F_n

$|E(F_n)| = 2n - 1$ and $|V(F_n)| = n + 1$

Define the function $f: E(F_n) \to \{0,1\}$ by following two cases.

Case 1: When n is even

$$f(e_i) = 0; \quad 1 \le i \le \frac{n}{2} \text{ and}$$

$$n+1 \le i \le \frac{3n}{2} - 1$$

$$f(e_i) = 1; \quad \frac{n}{2} + 1 \le i \le n \quad \text{and}$$

$$\frac{3n}{2} \le i \le 2n - 1$$

In view of the above defined labeling pattern, we have

$$v_f(0) = v_f(1) + 1 = \frac{n}{2} + 1$$

$$e_f(0) = e_f(1) - 1 = n - 1$$

Thus, we have $|v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$.

Case 2: When n is odd

In order to satisfy the edge condition for edge product cordial graph it is essential to assign label 0 to $\left\lceil\frac{2n-1}{2}\right\rceil$ edges out of $2n-1$ edges. The edge with label 0 will give rise atleast $\frac{n+3}{2}$ vertices with label 0 and atmost $\frac{n-1}{2}$ vertices with label 1 out of $n+1$ vertices. Therefore $|v_f(0) - v_f(1)| \ge 2$. Thus the vertex condition for edge product cordial graph is violated.

Hence the graph F_n is an edge product cordial graph for even n and not an edge product cordial graph for odd n.

Illustration 2.2 The fan F_4 and its edge product cordial labeling is shown in Figure 1

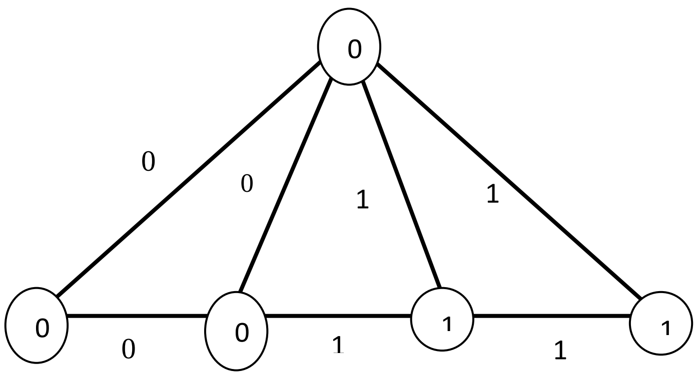

Figure 1

Theorem 2.3 For $q \ge 3$, the friendship graph $f_{q,p}$ is edge product cordial graph when q is odd.

Proof: Let $f_{q,p}$ be a friendship graph with odd q.

Let $e_1, e_2, \dots, e_{qp}$ be the successive edges of $f_{q,p}$ for odd q.

Define the function$f: E(f_{q,p}) \to \{0,1\}$ by the following two cases.

Case (i): when p is odd

$$f(e_i) = 1; \quad 1 \le i \le \frac{qp+1}{2}$$
$$f(e_i) = 0; \qquad otherwise$$

In view of the above defined labeling pattern, we have

$$v_f(0) = v_f(1) + 1 = \frac{(q-1)p}{2} + 1$$

$$e_f(0) + 1 = e_f(1) = \frac{qp+1}{2}$$

Case (ii): when p is even

$$f(e_i) = 1; \quad 1 \leq i \leq \frac{qp}{2}$$

$$f(e_i) = 0; \qquad otherwise$$

In view of the above defined labeling pattern, we have

$$v_f(0) = v_f(1) + 1 = \frac{(q-1)p}{2} + 1$$

$$e_f(0) = e_f(1) = \frac{qp}{2}$$

Thus in all cases we have $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Hence, the friendship graph $f_{q,p}$ is edge product cordial graph for all p when q is odd.

Theorem 2.4 $f_{q,p}$ is edge product cordial graph when q and p are even.

Proof: Let $f_{q,p}$ be a friendship graph with even q and even p.

Let $e_1, e_2, \ldots, e_{qp}$ be the successive edges of $f_{q,p}$ for even q and p

Define the function $f: E(f_{q,p}) \rightarrow \{0,1\}$ by

$$f(e_i) = 1; \quad 1 \leq i \leq \frac{qp}{2}$$

$$f(e_i) = 0; \qquad otherwise$$

By the above defined labeling pattern, we have

$$v_f(0) = v_f(1) + 1 = \frac{(q-1)p}{2} + 1$$

$$e_f(0) = e_f(1) = \frac{qp}{2}$$

Thus we have $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Hence, the friendship graph $f_{q,p}$ is edge product cordial graph when q is even and p is even.

Illustration 2.5 The graph $f_{4,4}$ and its edge product cordial labeling is shown in Figure 2

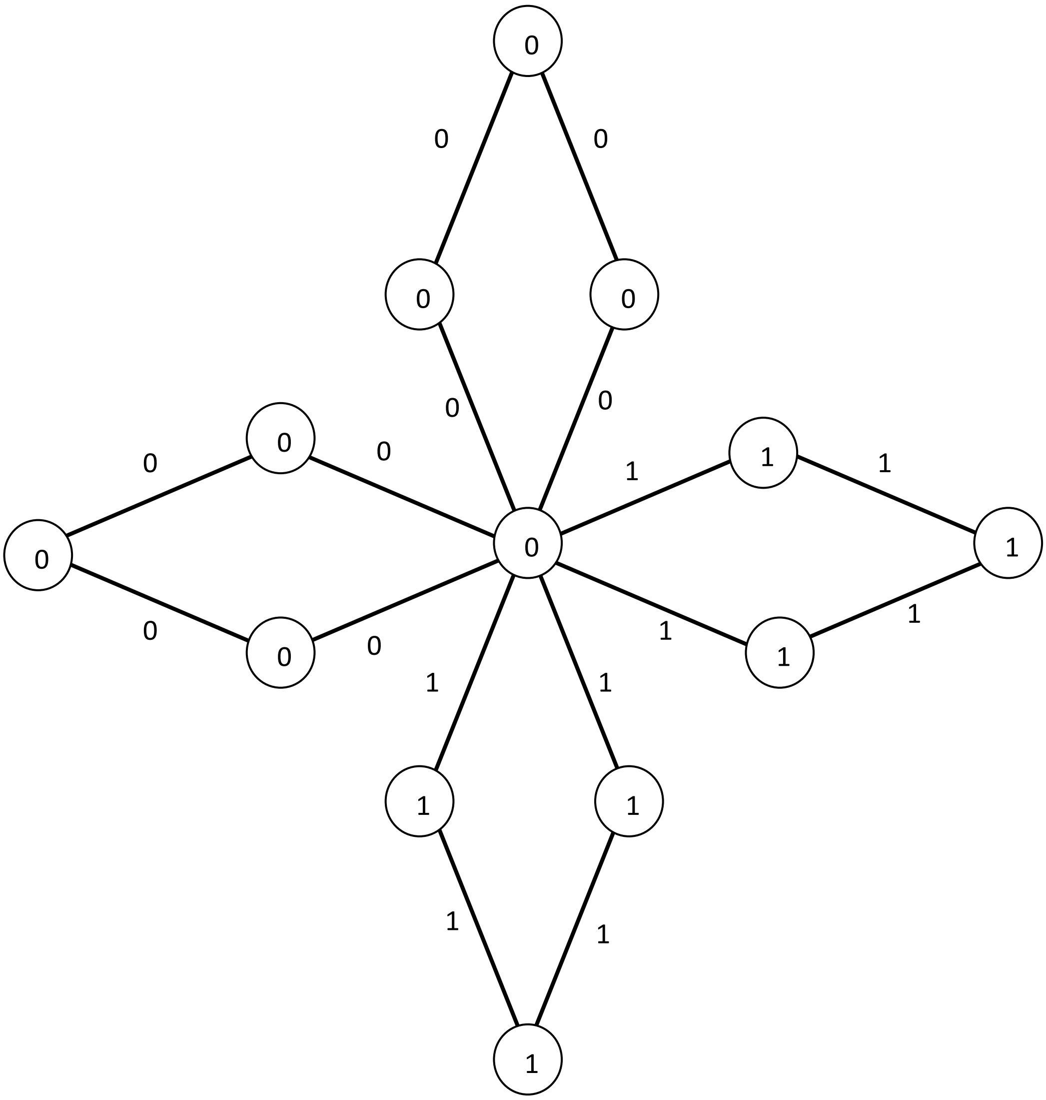

Figure 2

Theorem 2.6 The $n-$ sunlet graph admits edge product cordial graph.

Proof: Let S_n be a sunlet graph with $2n$ edges.

Let us denote the edges of the cycle C_n by $e_i$where $i = 1,2, \ldots, n$ and the pendant edges by e_j where $j = 1,2, \ldots, n$.

Define the function $f: E(S_n) \rightarrow \{0,1\}$ by

$$f(e_i) = 0 \; if \; i \cong 0,1 \; mod \; 2$$

$$f\left(e_j\right) = 1 \; if \; j \cong 0,1 \; mod \; 2$$

In view of the above defined labeling pattern, we have

$$v_f(0) = v_f(1) = n$$

$$e_f(0) = e_f(1) = n$$

Thus, we have $\left|v_f(0) - v_f(1)\right| \leq 1$ and $\left|e_f(0) - e_f(1)\right| \leq 1$.

Hence, $n -$ sunlet graph admits edge product cordial graph.

Example 2.7 The sunlet S_4 and its edge product cordial labeling is shown in Figure 3

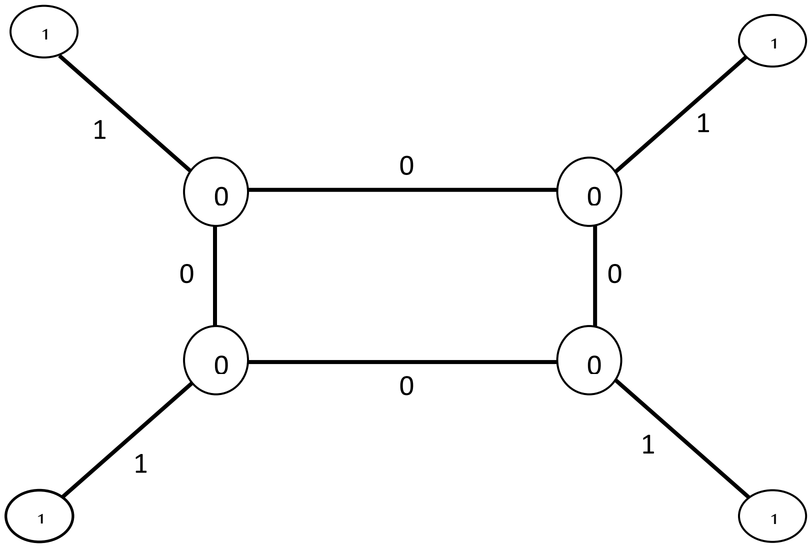

Figure 3

Theorem 2.8 The graph $T(n)$ is edge product cordial graph.

Proof: Let $e_1, e_2, \ldots, e_{n-1}$ be the edges of path P_n, $e'_1, e'_2, \ldots, e'_{n-2}$ be the upper pendent edges and $e''_1, e''_2, \ldots, e''_{n-2}$ be the lower pendent edges.

$|E(T_n)| = 3n - 5$ and $|V(T_n)| = 3n - 4$

Define $f: E\big(T(n)\big) \rightarrow \{0,1\}$ by following cases.

Case 1: when $n \equiv 0 \ (mod\ 3)$

$$f(e_i) = 0; for\ all\ i$$

$$f(e'_i) = 1; for\ all\ i$$

$$f(e''_i) = \begin{cases} 1\ ; 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor \\ 0\ ;\ \ otherwise \end{cases}$$

In view of the above defined labeling pattern, we have

$e_f(0) = \left\lfloor \frac{3n-5}{2} \right\rfloor$ and $e_f(1) = \left\lceil \frac{3n-5}{2} \right\rceil$

$v_f(1) = \left\lfloor \frac{3n-4}{2} \right\rfloor$ and $v_f(0) = \left\lceil \frac{3n-4}{2} \right\rceil$

Case 2: when $n \equiv 1\ (mod\ 3)$

$$f(e_i) = 0; for\ all\ i$$
$$f(e_i') = 1; for\ all\ i$$
$$f(e_i'') = \begin{cases} 1\ ; 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor \\ 0\ ;\ \ otherwise \end{cases}$$

In view of the above defined labeling pattern, we have

$e_f(0) = \left\lfloor \frac{3n-5}{2} \right\rfloor$ and $e_f(1) = \left\lceil \frac{3n-5}{2} \right\rceil$

$v_f(1) = \left\lfloor \frac{3n-4}{2} \right\rfloor$ and $v_f(0) = \left\lceil \frac{3n-4}{2} \right\rceil$

Case 3: when $n \equiv 2\ (mod\ 3)$

$$f(e_i) = 0; for\ all\ i$$
$$f(e_i') = 1; for\ all\ i$$
$$f(e_i'') = \begin{cases} 1\ ; 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor \\ 0\ ;\ \ otherwise \end{cases}$$

In view of the above defined labeling pattern, we have

$e_f(1) = \left\lfloor \frac{3n-5}{2} \right\rfloor$ and $e_f(0) = \left\lceil \frac{3n-5}{2} \right\rceil$

$v_f(1) = \left\lfloor \frac{3n-4}{2} \right\rfloor$ and $v_f(0) = \left\lceil \frac{3n-4}{2} \right\rceil$

Thus in all cases we have $\left|v_f(0) - v_f(1)\right| \leq 1$ and $\left|e_f(0) - e_f(1)\right| \leq 1$.

Hence, the graph $T(n)$ is edge product cordial graph.

Illustration 2.9 The graph T_5 and its edge product cordial labeling is shown in Figure 4.

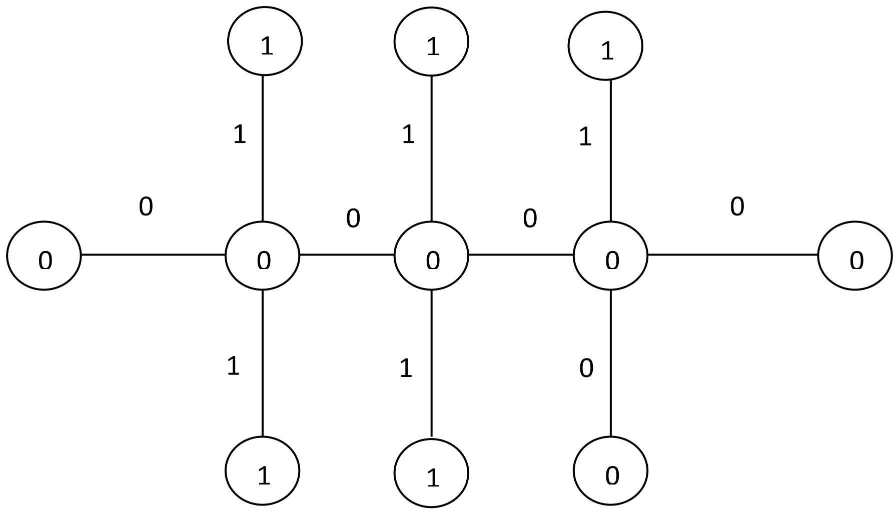

Figure 4

Theorem 2.10 The graph Hd_n is edge product cordial graph.

Proof: Let $e_1, e_2, \dots, e_{n-1}$ be the edges of path P_n and $e'_1, e'_2, \dots, e'_{n-2}$ be the pendant edges .

$|E(T_n)| = 2n - 3$ and $|V(T_n)| = 2(n-1)$

Define $f: E(Hd_n) \to \{0,1\}$ by

$$f(e_i) = \begin{cases} 1\,; & i = 1 \\ 0\,; & otherwise \end{cases}$$

$$f(e'_i) = 1; \qquad for\ all\ i$$

In view of the above defined labeling pattern, we have

$$v_f(0) = v_f(1) = n - 1$$

$$e_f(1) = e_f(0) + 1 = n - 1$$

Thus we have $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Hence the graph Hd_n is edge product cordial graph.

Example 2.11 The graph Hd_6 and its edge product cordial labeling is shown in Figure 5

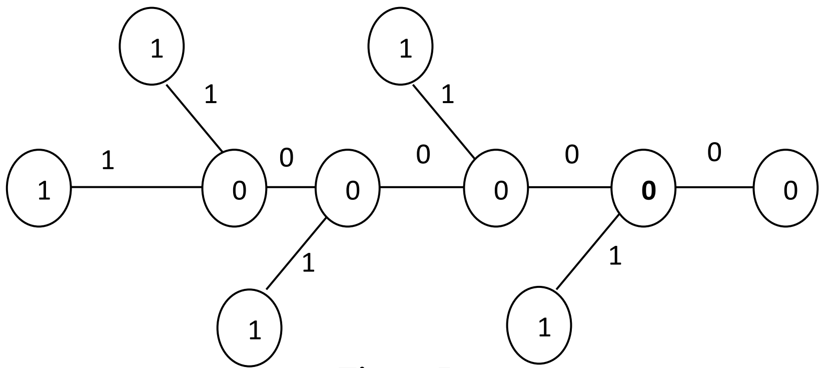

Figure 5

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